

The Chain Rule

Chain Rule is a method used to find the derivative of a function that has other functions inside itself (when you have the composition of functions).

$$\textcircled{X}^5 \rightarrow y' = 5\textcircled{X}^4 \cdot 1 [f(g(x))] = f'(g(x))(g'(x))$$

$$(4x^5 + 2x^2)^5 \quad \left. \begin{array}{l} y' = 5(4x^5 + 2x^2)^4 (20x^4 + 4) \\ y' = 6(6x^3 + 5x)^5 (18) \end{array} \right| \quad \begin{array}{l} 2) y = (6x^3 + 5x)^6 \\ y' = 6(6x^3 + 5x)^5 (18) \end{array}$$

$$\sqrt{x^3 + 7} = (x^3 + 7)^{1/2} \quad \left. \begin{array}{l} y' = \frac{1}{2}(x^3 + 7)^{-1/2} (3x^2) \\ y' = \sec^2(6x) \cdot 6 \end{array} \right| \quad \begin{array}{l} 4) y = \tan 6x \\ y' = \sec^2(6x) \cdot 6 \end{array}$$

$$\sin^4 x = (\sin x)^4$$

$$6) f(x) = \sin(x)$$

$$y' = 4(\sin^3 x) \cdot \cos x$$

$$f'(x) = \cos(x^4) \cdot 4$$

$$x^2 \cdot x = x^3$$

$$= \sqrt{\sin(6x)} = (\sin(6x))^{1/2}$$

$$y' = \frac{1}{2}(\sin(6x))^{-1/2} \cdot \cos(6x) \cdot 6$$

$$y' = 3(\sin(6x))^{-1/2} \cdot \cos(6x)$$

$$8) y = \sec^3(2x)$$

$$y' = 3(\sec^2(2x)) \cdot (\sec(2x))'$$

$$y' = 6 \sec^3(2x) \cdot \tan(2x)$$

$$x^6)(3x^2 + x)^4$$

$$10) \quad y = (3x^3 + 5)^3 (4x)$$

$$5x^5(3x^2 + x)^4 + (5x^6) \cdot 4(3x^2 + x)^3(6x+1)$$

$$5x^5(3x^2 + x)^4 + 20x^6(3x^2 + x)^3(6x+1)$$

$$(3x^3 + 5)^2(9x^2)(4x^2 + 2x) + (3x^3 + 5)^3 \cdot (8x + 2)$$

$$7x^2(3x^3 + 5)^2(4x^2 + 2x) + (3x^3 + 5)^3 \cdot (8x + 2)$$

$$\frac{3x^4}{(6x^3 + x)^2}$$

$$\cancel{x+x} \cdot 12x^3 - 3x^4 \cdot 2(6x^3 + x)(18x^2 + 1)$$

$$(6x^3 + x)^{4-3}$$

$$\underline{x^3(6x^3 + x) - 6x^4(18x^2 + 1)}$$

$$(6x^3 + x)^3$$

$$\tan^3 x \cot^4 x$$

$$\cancel{\tan^2 x} \cdot 3\sec^2 x \cdot \cot^2 x + \cancel{\tan^3 x} \cdot 4\cancel{\cos^5 x} \cdot (-\csc^2 x)$$

$$3\sec^2 x \cdot \cot^2 x - 4\csc^2 x$$

$$\frac{1}{\cancel{\cos^5}} \frac{\cancel{\cos^4}}{\sin^2}$$

$$y' = 3\csc^2 x - 4\csc^2 x = -1\csc^2 x = \boxed{-\csc^2 x}$$

$$12) f(x) = (3x^5) \cos$$

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in 2
slides

$$14) k(x) = \frac{2x^4}{\cos^5 x}$$

$$= \frac{2}{(5x+1)^2}$$

$$)=\left(3x^5 \right) \cos ^6 x$$

$$\therefore f'(x)=15x^4 \cdot \cos ^6 x+3x^5 \cdot (6 \cdot \cos ^5 x \cdot (-\sin x))$$

$$f'(x)=15x^4 \cos ^6 x-\left(18x^5 \right) (\cos ^5 x)(\sin x)$$

$$14) \quad k(x) = \frac{2x^4}{\cos^5 x}$$